

Capacity of a Cognitive Radio Channel with Rayleigh Distribution in Presence of Noise and one Interfering Signal

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Abstract—Most of the work done in cognitive radio has been done assuming a Gaussian channel. In this paper a Rayleigh channel has been assumed and the effect on the channel capacity is observed when the Rayleigh scale parameters of signal and interference are changed. It discusses interference in CRNs using the concept of interference aware spectrum sensing. In this paper one interfering signal has been assumed along with the noise which has been taken to be constant. The interfering signal also has a Rayleigh distribution. Keeping these things in mind, the Ergodic capacity of the channel has been calculated after calculation of the probability density function of each of the terms involved viz. signal power, noise and interference power and the signal to interference and noise ratio, SINR. The effect on the capacity with the change in the scale parameter of the Rayleigh distribution, both of signal and interfering signal is observed.

1. INTRODUCTION

In these times, due to the focus on higher and higher data rates, no amount of spectrum seems enough. Cognitive radio is a breakthrough technology which is in a position to alleviate this spectrum scarcity problem. Most of the work done in cognitive radio has been done using Gaussian channel [1] but in this paper, Rayleigh channel has been assumed for signal, interference and noise. Here the focus is on interference aware spectrum sensing in presence of noise [2] where the system works in the presence of interference assuming that primary and secondary users are using the channel simultaneously. In this paper one interfering channel has been assumed which is causing interference to the primary signal in presence of noise. If secondary continues or starts transmission during primary transmission, then it is a case of missed detection [3] which happens if the spectrum sensing does not detect the primary signal when the secondary is transmitting. Since the spectrum sensing is to be performed by the secondary, missed detection is a failure on part of the secondary as interfering primary transmission is to be avoided at all costs [4]. The foundation of cognitive radio is based on the premise that the unlicensed (secondary) user can use the spectrum of the licensed (primary) user only if the primary transmission is not

interrupted or interfered with by the secondary. Things have slightly changed since the advent of cooperative sensing [3,5,6] where the possibilities of missed detection and false alarm are reduced considerably by cooperative sensing of the spectrum by a group of secondary users which collectively use the spectrum of the licensed user. Simultaneous transmissions by the primary and secondary are also a possibility now under certain laid down conditions as in the case of underlay systems designed for cognitive radio where both primary and secondary users can use the spectrum together if the secondary can keep its signal power below a certain threshold so as not to interfere with the primary transmission. For considering interference aware spectrum sensing in the cognitive radio transmission, missed detection is a pre requisite [7]. In this paper we consider both primary and secondary users transmitting simultaneously and secondary not transmitting below the specified threshold power limit due to which the primary is experiencing interference from secondary. The effect this interference will have on its Ergodic capacity when the Rayleigh scale parameters are varied is what will be studied in this paper.

2. SYSTEM MODEL

In this paper, for spectrum sensing energy detection [8] is used. The signal $s(n)$ is transmitted over a fading channel whose gain is $h(n)$ and then corrupted by the noise $w(n)$ where n is the sample index. In this case, the detection problem is formulated as

$$\begin{aligned} H_0 : x(n) &= w(n) \\ H_1 : x(n) &= h(n)s(n) + w(n) \end{aligned} \quad \dots \dots (1)$$

where H_0 and H_1 indicate the absence and presence of primary signal respectively as a result of spectrum sensing.

The decision metric M for the energy detector can be written as

$$M = \sum_1^N |x(n)|^2 \quad (2)$$

The decision on band occupancy can be arrived at by comparing the decision metric M against a fixed threshold λ_E .

The detection algorithm in cognitive radio can be summarized by two probabilities [9], probability of detection, P_d (when the primary signal is actually present in the given channel) and probability of false alarm, P_f (when the primary signal is detected but is not actually present). These probabilities are expressed as under.

$$P_d = \Pr (M > \lambda_E | H_1) .$$

$$P_f = \Pr (M > \lambda_E | H_0) \quad \dots(3)$$

The signal $x(n)$ is the modulated signal. The channel gain $h(n)$ follows an independent and identically distributed (i.i.d.) Rayleigh fading channel, i.e. $h(n) \sim CN(0, \sigma^2)$. The noise is taken to be constant

The probability density function of the received signal is calculated next [10] using Rayleigh distribution and ignoring the noise. This comes out to be

$$f_X(y) = \frac{-y}{e^{2\sigma^2}} \quad \dots(4)$$

where $f_X(y)$ is the pdf of the signal, σ is the scale parameter for the Rayleigh distribution and y is the signal variable. Fig. 1 shows the graph for it's probability density function, pdf.

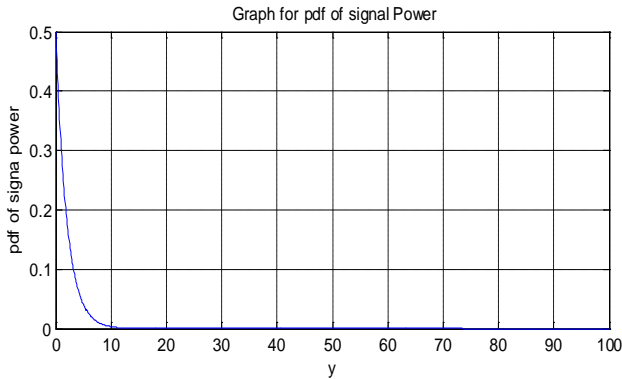


Fig. 1: pdf of Signal power

The sum of the interference power and noise power needs to be considered next and it is equal to $(x_i^2 + N_0)$, where x_i^2 is the interference power and N_0 the noise power density which is taken to be constant. The probability density function of the sum is calculated next [10], again using Rayleigh distribution. This comes out to be as under and is shown in Fig. 2.

$$f_Y(y) = \frac{1}{2\sigma_i^2} e^{\frac{-(y-N_0)}{2\sigma_i^2}} \quad \dots(5)$$

The SINR, signal to noise and interference ratio is given by

$$\gamma = f_X(y)/f_Y(y) \quad \dots(6)$$

and its pdf [10] , shown in Fig. 3, is calculated to be

$$f_Y(Y) = \left(\frac{-\gamma N_0}{e \frac{2\sigma^2}{4\sigma_i^2 \sigma^2}} \right) \left(1 - N_0 \left(\frac{\gamma}{2\sigma_i^2} + \frac{1}{2\sigma^2} \right) \right) \left(\frac{\gamma}{2\sigma_i^2} + \frac{1}{2\sigma^2} \right)^2 \quad \dots(7)$$

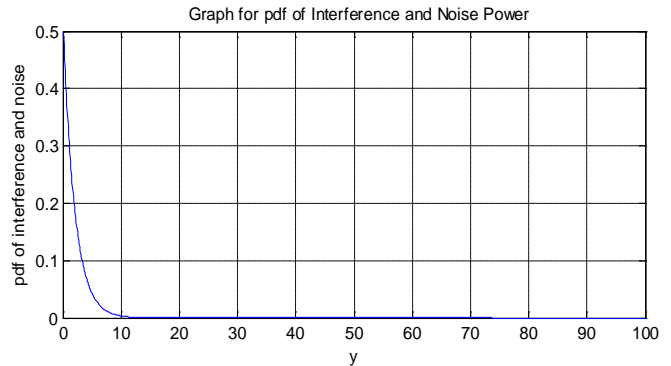


Fig. 2: pdf of Noise and Interference Power

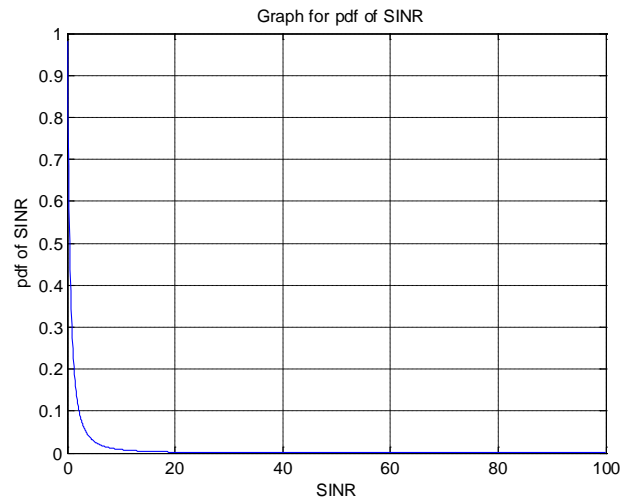


Fig. 3: pdf of SINR

On integration, all the pdf's are found to equal 1 as shown in Fig. 4 for SINR only, and was found true for all other pdf's also, thereby authenticating the equations.

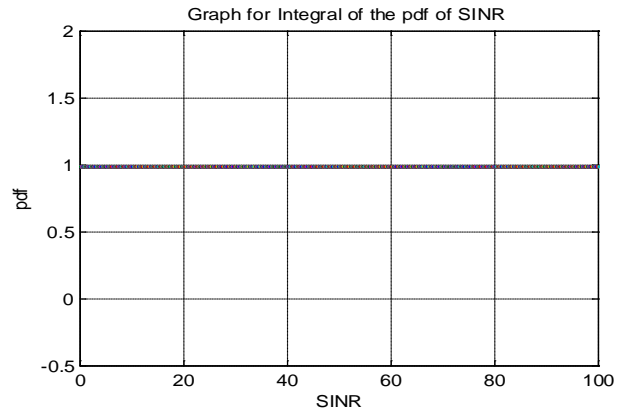


Fig. 4: Integral of pdf of SINR

3. CAPACITY

The Ergodic capacity of the channel is calculated next. The Ergodic capacity [11] for a fading channel is given by the expression.

$$C = B \int_0^{\infty} \log_2(1 + \gamma) f_{\gamma}(\gamma) d\gamma \quad (8)$$

which gives the average value of the channel capacity of a fading channel, the instantaneous channel capacity of which is given by

$$C = B \log_2(1 + \gamma) \quad (9)$$

where B is the bandwidth, C is the Ergodic channel capacity and the rest of the terms remaining the same in meaning as before. This can also be written in terms of capacity per unit bandwidth as

$$C/B = \int_0^{\infty} \log_2(1 + \gamma) f_{\gamma}(\gamma) d\gamma \quad (10)$$

After substitutions this expression reduces to

$$\frac{C}{B} = \int_0^{\infty} \log_2(1 + \gamma) \left(\frac{\frac{-\gamma N_0}{e 2\sigma^2 g}}{4\sigma_i^2 \sigma^2} \right) \left(\frac{1 - N_0 \left(\frac{\gamma}{2\sigma_i^2} + \frac{1}{2\sigma^2} \right)}{\left(\frac{\gamma}{2\sigma_i^2} + \frac{1}{2\sigma^2} \right)^2} \right) d\gamma \quad (11)$$

Now if we consider average SINR ($\bar{\gamma}$) in place of (γ), we get the final expression as

$$\frac{C}{B} = \int_0^{\infty} \log_2(1 + \gamma) \left(\frac{\frac{-\gamma N_0}{e 2\sigma^2 g}}{4\sigma_i^2 \sigma^2} \right) \left(\frac{1 - N_0 \left(\frac{\gamma}{2\sigma_i^2} + \frac{1}{2\sigma^2} \right)}{\left(\frac{\gamma}{2\sigma_i^2} + \frac{1}{2\sigma^2} \right)^2} \right) d\gamma \quad (12)$$

where $g = 10^{(\bar{\gamma}/10)}$, and ($\bar{\gamma}$) is the gammabar or average SINR.

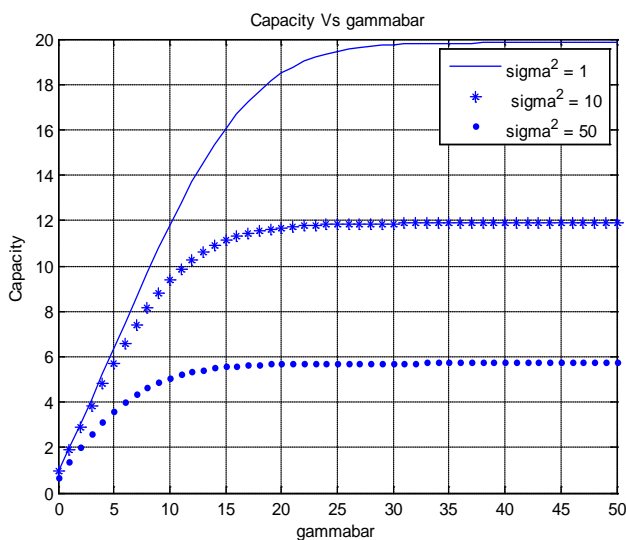


Fig. 5: Graph for Capacity Vs average SINR for different values of σ_i^2

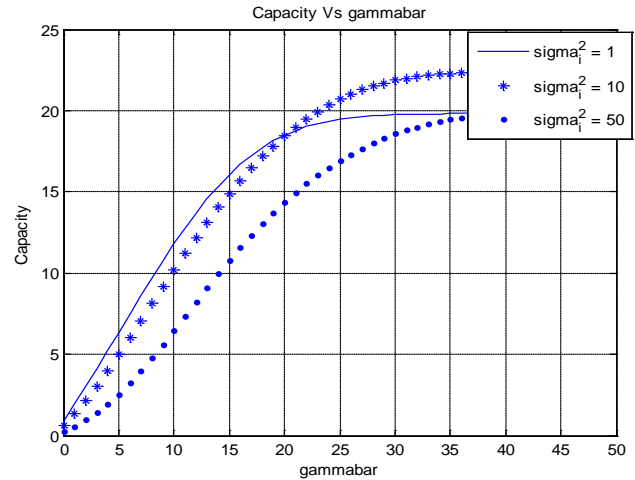


Fig. 6: Graph for Capacity Vs average SINR for different values of σ^2

By substituting different values for σ_i^2 and σ^2 , different graphs for Capacity vs $\bar{\gamma}$ (gammabar) can be plotted as shown in Fig. 5 and Fig. 6.

4. CONCLUSION

The work done in this paper is based on one interferer to the primary signal in presence of the noise. The channel considered here is having Rayleigh distribution for the signal as well as the interference and noise. The Ergodic channel capacity has been calculated for this channel. The effect of the different values of σ_i^2 and σ^2 , the scale parameters of Rayleigh channel for interference and signal respectively, on the channel capacity has been observed. In future work, more than one interfering signal to the primary user's signal will be considered along with the noise and the effect on Ergodic channel capacity and outage capacity will be considered. That work may be done considering Rayleigh fading channel or Nakagami-m channel.

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